THERMAL EFFECTS ON CARBON BASED ELECTRODES CLOSE TO A HIGH CURRENT ELECTRIC ARC

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ABSTRACT

The electrodes of a Submerged Arc Furnaces carry a high current and are subject to enormous stresses caused by both the electric arc itself, and the current distribution within the electrode. The external effect of the arc on the electrode is treated in a different paper at this conference, The purpose of this paper is to analyze the thermal effects of ohmic heating within the electrode at the arc attachment on the carbon based electrode, using data for Søderberg electrodes.. The thermal conditions are then used to examine stress conditions in the vicinity of an arc.

Two types of models of an electrode are presented in the paper. The first type is an accurate finite element model, based on both two and three dimensional elements and the second type is an one dimensional model, providing better insight into the problem. For the second type, a spherical model of thermal conditions in a semi-infinite solid has been developed.

Results show that the electrode surface is heated quickly up to a maximum temperature and for a specific test case, the surface can only withstand the load for about 10 seconds. Also, the thermal stresses on the surface are very high which will increase erosion on the surface in addition to normal evaporation.

1. INTRODUCTION

Ferrosilicon is most frequently produced in submerged arc furnaces with Søderberg electrodes, see [1]. The metal is produced in a chemical process which receives energy from an electric arc. Carbon based electrodes are needed for that purpose. The uniqueness of the Søderberg process originates from the fact that these electrodes are manufactured during and in connection with the production itself.

In order to understand better the part of the process that has to do with the energy flow into the chemical process taking place in the submerged arc furnace, fairly accurate mathematical models have been derived. The focus in most of these models has been the electrode, see e.g. [2], as well as the arc, see [3], since operating electrodes is a critical process in the furnace operation.. In order to grasp the fundamental principles, a simple one-dimensional model was presented in [4] with the main purpose of describing the phase transition at the arc surface. A more complex two dimensional model was furthermore used in [5] where the focus is on conditions that prevail in the initiation phase of a cathode spot. The results of this work indicates an overheating area of the spot, inside the surface itself, due to ohmic heating and heat losses at the surface.

The purpose of this paper is to focus on the electrode near the spot where the electric current leaves the electrode and forms an electric arc. This can be done with a relatively simple heat transfer model, similar to the model in [4], where a solid with alternating current flowing through it is modelled. The solid is semi-infinite and therefore only one surface (or boundary) is included in the analysis. The current flows out of the solid at the surface at one single area, which is circular with a given radius. Therefore it is assumed that the current density in the whole solid is pointed at this single area, which simplifies the problem.

The problem is furthermore analyzed by using an axi-symmetric finite element model in two dimensions. This is done primarily in order to verify the one dimensional model, and also to compute structural stresses in the arc vicinity.

This particular problem definition corresponds to a conductor with a single electric arc burning from the circular area as shown in figure 1. The figure also shows the model domain near the arc.

The paper is organized as follows. In section 2 a mathematical model is posed, which describes the problem. This is followed by some general results in section 3 and the by a real case study in section 4. Finally, conclusions are drawn in section 5.

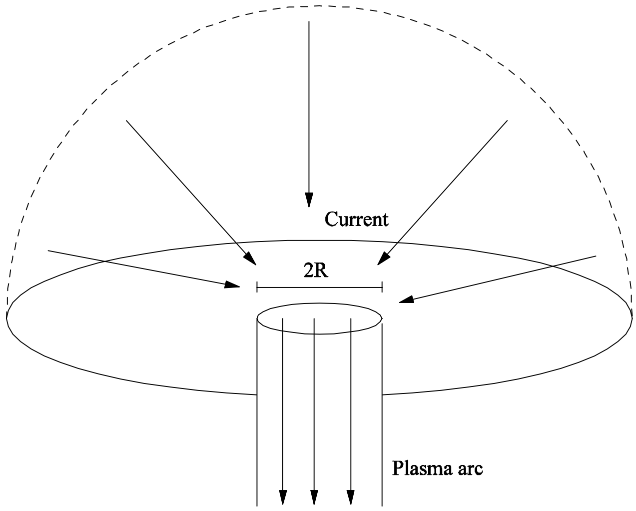


Figure 1: Semi-infinite solid with an electric arc on the surface

2. MATHEMATICAL MODELS

2.1 The one dimensional case

A dynamic representation of the temperatures in the solid is described mathematically as an initial value problem. A general formulation of this problem is

$$\rho c \frac{\partial T}{\partial t} = k\nabla^2 T + \frac{J^2}{\sigma} \tag{1}$$

where ρ is density, c is specific heat, k is thermal conductivity, J is current density, σ is electric conductivity and T is the temperature.

The general problem can be simplified considerably by assuming spherical symmetry in the problem. This implies the following:

- All variables and parameters are constant at a given distance from the center of the arc area.
- It is assumed that a total current I flows directly towards the center and is uniform at a fixed distance.
- A spherical hole is assumed to be present at the arc center, so the solid is bounded by a half-sphere with radius R_s.
- The surface is insulated if the distance from the center is greater than R_s.

These assumptions allow for a model in spherical coordinates where the radius r is the only spatial variable. If the total current is given as I, then the current density becomes $J = \frac{I}{2\pi r^2}$ as it goes through a half-sphere. The initial value problem for the heat equation (1) then becomes

$$\rho c \frac{\partial T}{\partial t} = \frac{k}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{I^2}{4\pi^2 \sigma r^4} \tag{2}$$

2.2 Dimensionless form of the heat equation

It is convenient to use dimensionless variables in (2) where it is possible. Consequently the following dimensionless variables are defined

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty} \qquad \eta = \frac{R_s}{r} \qquad \tau = \frac{k}{\rho c R_s^2} t$$

where T_0 is a reference temperature at the boundary and T_∞ is the temperature far away from the center (when $r \to \infty$).

The spatial differential operator is transformed into the dimensionless variable η and the time variable becomes τ. Therefore the differential operators become

$$\frac{\partial}{\partial r} = -\frac{\eta^2}{R_s}\frac{\partial}{\partial \eta} \qquad \text{and} \qquad \frac{\partial}{\partial t} = \frac{k}{\rho c R_s^2}\frac{\partial}{\partial \tau}$$

When τ and η are inserted into (2) the result is

$$\eta^{-4}\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + 2\beta \tag{3}$$

where

$$\beta = \frac{I^2}{8\pi^2 \sigma k (T_0 - T_\infty) R_s^2} \tag{4}$$

It is clear from (3) and (4) that all the physical properties of the problem can be gathered into a single parameter β. This simplifies further analysis of the problem, since the solution to the partial differential equation in (3) can be developed as a single parameter function, which includes all the characteristics of the physical problem.

2.3 Finite element modeling

In order to compare the one dimensional model to a more realistic case, a finite element representations of an electrode has been built.

A two dimensional axi-symmetric model is considered, where it is assumed that the current flows from the electrode bottom. The accuracy of this model can be increased by using a very dense mesh, and it is applecible for both static and dynamic analysis.

3. ONE DIMENSIONAL ANALYTICAL RESULTS

3.1 A steady state solution

An extreme case of an arc flowing out from a surface is where the arc does not move, and thus all time changing conditions converge to a steady state solution. This corresponds to defining $\frac{\partial\theta}{\partial\tau}=0$ and thus the equation to be solved is

$$\frac{\partial^2\theta}{\partial\eta^2}+2\beta=0$$

By assuming the boundary conditions θ(1) = 1 and τ(0) = 0, corresponding to $T(R_s) = T_0$ and $T(\infty) = T_\infty$, the solution becomes

$$\theta(\eta)=\eta(1+\beta(1-\eta))$$

Some interesting values can be derived from the solution. The first one is the maximum value of and the position of that value. Differentiation gives

$$\eta_{\max}=\frac{1}{2}+\frac{1}{2\beta} \qquad \text{and} \qquad \theta_{\max}=\frac{(1+\beta)^2}{4\beta}$$

Another interesting value is the maximum value of η, where $\theta\le 1$. This corresponds to the minimum radius *r* where $T\le T_0$. The solution to this problem is

$$\eta=\frac{1}{\beta} \tag{5}$$

which corresponds to the intrusion of temperatures higher than T_0 in to the solid. The assumption of $\theta\le 1$ is justified by realizing that the maximum temperature of the material is $\theta=1$, since it will be sublimed into gas at higher temperatures. Values of $\theta>1$ are therefore outside the physical frame of the problem.

3.2 A time dependent solution

It is clear that the steady state solution can only be used as a reference since a realistic case study would not have an arc at the same spot at all times. Therefore it is necessary to solve the time dependent problem and thus solve (3) with appropriate boundary conditions.

It turns out that (3) is hard (or impossible) to solve analytically, and numerical methods must be used. This is done by discretizing the space derivative with finite differences and then integrating the result as a system of ordinary differential equations.

The discretization is performed by dividing the interval $0\le\eta\le 1$ into *N* spaces, such that θ_j is an approximation of θ for *j* = 0 ... *N*. The result is a set of equations

$$\frac{\partial\theta_j}{\partial\tau}=\eta^4N^2\left(\theta_{j-1}+\theta_{j+1}-2\theta_j\right)+2\eta^4\beta \tag{6}$$

which are mutually dependent. This is then integrated by using a numerical method, in this case a Runge-Kutta method for stiff systems.

The solution of (6) is dependent on β and therefore there is not much point in plotting it as a general solution. But there are some derived quantities that are of special interest and can be shown as a single graph with respect to β.

The first study involves a problem where the initial condition $\theta(\eta,0)=0$ is applied, as well as the boundary conditions $\theta(0,\tau)=0$ and $\theta(1,\tau)=1$. Thus the temperature at the boundary is raised suddenly to T0, which is what happens if an arc is suddenly started at the surface. It is of interest to find out how long time passes until this surface temperature starts penetrating the surface (and thus generating abnormally high temperatures inside the solid). Figure 2 shows this relationship for a wide range of β.

It can finally be concluded that figure 2 shows that the time until penetration occurs decreases with increasing arc current I and with decreasing electric conductivity of the electrode material.

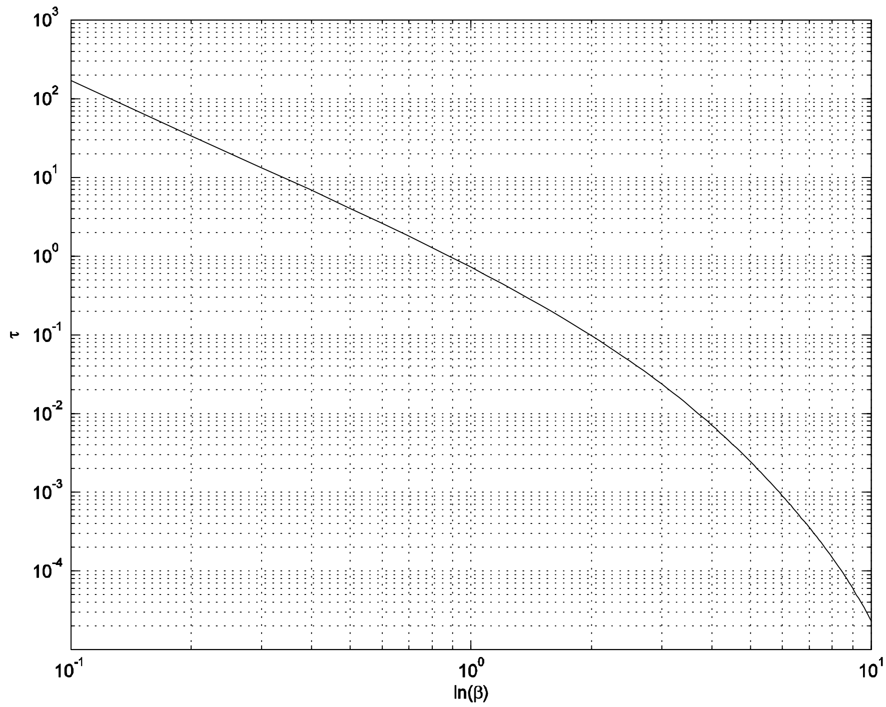


Figure 2: Dimensionless time until temperature penetration occurs as a function of ln(β)

4. A REAL WORLD CASE STUDY

A realistic case study is analysed in this section, where a baked carbon paste of a Søderberg electrode is considered. Real material parameters are used and the applied current is assumed to be I = 115 kA. It is further assumed that the sublimation temperature is T_0 = 4000 °C and the environment is at T_∞ = 1000 °C. For a discussion of material parameters in Søderberg electrodes see e.g. [7] and [8].

4.1 The one dimensional model

An one dimensional static model was implemented in order to get an overview of what is important at the arc attachment, and how large a role ohmic heating plays at the arc interface. It is assumed that the arc has a radius of R = 5 cm and therefore a natural choice would be to set $R_s = R$. But it should be noted that the surface area of the half-sphere bounded by R_s is not the same as the surface are of an arc flowing from the bottom. Therefore, a correction is also considered which results in $R_s = R / \sqrt{2}$ = 3.54 cm. Both these cases are examined and the results are shown in table 1.

Apparently there is a considerable difference between the two chosen values of the radius, so a comparison to a more complete finite element model should indicate which method should be used.

Even though the heat flux from the arc itself is discounted, and the only source of heating is the electric heating, the surface that carries the sublimation temperature has a distance of the order of 1m. from the arc surface. This indicates that the arc/electrode system can never be static. As the carbon material in the electrode reaches the sublimation temperature, a geyser of carbon vapour will drive the arc away. It is estimated that the maximum time for a arc spot to be static is around 10 seconds, as seen in Table 1.

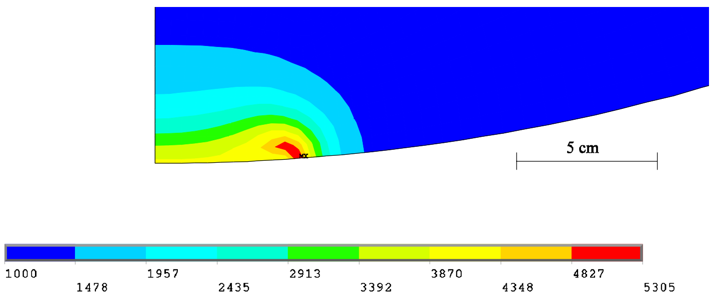
Table 1: Results for an one dimensional model

Result	*R = 5 cm*	*R = 3.54 cm*
Static R_{max} (based on η_{max})	9.5 cm	9.7 cm
Static T_{max} (based on θ_{max})	16500 °C	30400 °C
Static penetration of T_0 (based on (5))	0.93 m	1.86 m
Time until penetration starts	10.5 s	2.2 s

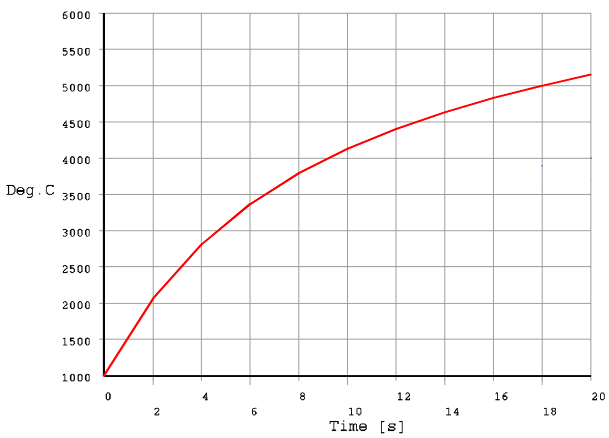
4.2 Time dependent results from an axi-symmetric finite element model

A model was constructed with the purpose of focusing on the electrode bottom and compare time dependent simulation to the results from the one dimensional model. It is assumed here that the current flows from the bottom in a circular pattern with a radius of 5 cm.

Figure 3(a) shows the temperature near the arc after 10 s of simulation time. Here it is clear that the temperature is in some points larger than T_0, but this can be analysed further by choosing a point near the arc and plotting the temperature as a function of time. This is shown in figure 3(b) where the maximum temperature is reached at time from about 9 s to 20 s. This is close to the result found with the one dimensional model where R_s = 5 cm.



(a) Distribution after 10 s



(b) As a function of time

Figure 3: Temperature in an axi-symmetric model [ºC]

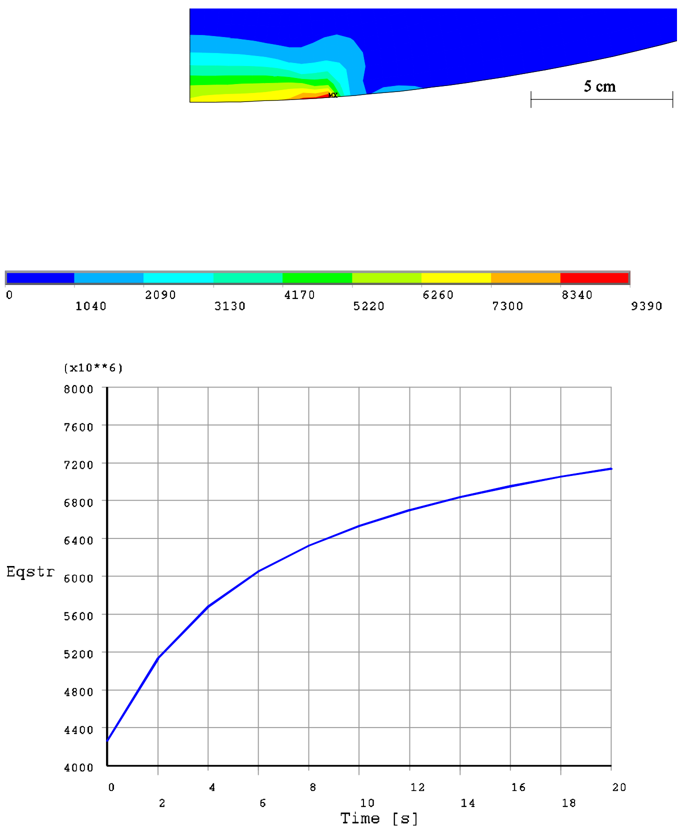
It should be noted that the edge of the arc (the outer surface for a circular arc) is a problematic spot for the finite element solution because of the fact that the current is forced to flow out perpendicularly to the surface. Nevertheless, this only affects the solution near the problem spot.

Therefore, by assuming that the current flowing into the arc is spread over a area with a 10 cm diameter, and disregarding the heat flux from the arc itself, the results from these simulations indicate that the arc can not burn in the same spot for more than about 10 seconds.

4.3 Structural stresses in an axi-symmetric model

The final study here is to look at structural stresses which are the result of temperature changes in the electrode. As before, it is assumed that this effect mostly occurs near the arc, and the results support this fact.

Figure 4(a) shows the stresses at the electrode bottom at a given time points. This time is the same as in figure 3(a), which is 10 s after the start of the arc at the bottom. It is clear that the stresses are closely related to the temperatures, and appear to be very high, several magnitudes higher than the stress that the electrode is supposed to withstand. This is the case in the contour region, but only penetrated about 2 cm into the electrode.



(a) Distribution after 10 s

(b) As a function of time

Figure 4: Equivalent stresses near the arc outlet in an axi-symmetric electrode [MPa]

Finally, figure 4(b) shows the changes in stresses with respect to time. This follows the change in temperatures very well, as expected.

5. CONCLUSIONS

A spherical model of thermal conditions in a semi-infinite solid is presented in this paper. This model is in reality one dimensional since changes are only considered in the radial direction. Furthermore, more complex finite element models have been built and compared to the one dimensional model.

The model seems to agree with the more complex two and three dimensional models, and is applicable in the vicinity of an electric arc. This indicates that the results found in figure 2 can be used to analyze the time dependent effect of an arc, burning at the bottom of an electrode. The key parameter here is β, which was defined in (4).

The results indicate that when an arc is formed with a relatively high current, then it affects the surface of the electrode greatly, both with respect to temperatures and structural stresses. Particularly, it is apparent that an electric arc with a current over 100 kA flowing from a Søderberg electrode may burn at the same point for no longer than 10 s, since the material cannot withstand higher temperatures than 4000 °C. It is then likely that the arc is moving around within such a time frame, or that more than one arc is present at once.

Furthermore, analysis of the thermal stresses due to the electric heating near the surface, disregarding heat flux from the arc, show very high stresses, which indicates that the mechanical erosion at the surface would considerably contribute to erosion of the electrode. A justification for disregarding the heat flux from the arc itself is the simple fact that the electrode material does not conduct heat very efficiently, and the heat transport due to conduction is of the order of 20 kW, as compared to the several MW delivered by the arc. The role of the arc itself on the electrode consumption is discussed in another paper at this conference.

The combined findings of this paper are an important addition to research on arc movement and electrode consumption. Further work could focus on the somewhat complex plastic behaviour of the electrode material.

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